(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 7304 **Roll No.**

M.C.A.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2010-11

DISCRETE MATHEMATICS

Time : 3 Hours *Total Marks : 100*

Note : Question paper carries three sections. Read the instructions carefully and answer accordingly.

SECTION—A

1. Attempt **all** parts of this section :

(i) This question contains 10 multiple choice questions. Select the correct answer for each one as per instruction :— **(10×1=10)**

(a) If A and B are two sets, then $A \cap (A \cup B)$ equals

(i) A (ii) B

(iii) ϕ (iv) None of these

(b) Let $f : R \rightarrow R$ be defined by $f(x) = x^2 + 1$, then $f^{-1}(5)$ is

(i) {–2, 2} (ii) {–3, 3}

(iii) {2, 2} (iv) {3, 3}

(c) If G is a finite group and H is a normal subgroup of G, then O(G/H) is equal to

(i) O(G) (ii) O(H)

(iii) O(G)/O(H) (iv) None of these

(d) The set of all positive rational numbers forms an abelian group under the composition defined by $a * b = \frac{ab}{2}$. Identity of this group is

(i) 1 (ii) 2

(iii) 0 (iv) None of these

(e) The chromatic number of any nontrivial tree is

(i) 1 (ii) 2

(iii) 4 (iv) None of these

(f) In how many ways can three boys and two girls sit in a row ?

(i) 48 (ii) 24

(iii) 120 (iv) None of these

(g) A vertex of degree zero is called

(i) Null vertex (ii) Isolated vertex

(iii) Proper vertex (iv) None of these

(h) Which of the following statement is tautology ?

(i) $(p \vee q) \Rightarrow q$ (ii) $p \vee (q \Rightarrow p)$

(iii) $p \vee (p \Rightarrow q)$ (iv) $p \rightarrow (p \Rightarrow q)$

(i) A logic gate is an electronic circuit which

(i) makes logic decisions

(ii) allows electron flow only in one direction

(iii) works on binary algebra

(iv) alternates between 0 and 1 values

(j) A complete n-ary tree is one in which every node has o or n sons. If x is number of internal nodes of complete n-ary tree, the number of leaves in it is given by

(i) $x(n-1)+1$ (ii) $xn-1$

(iii) $xn+1$ (iv) $x(n+1)$

(ii) State True or False : **(5×1=5)**

(a) The mapping $f : R \rightarrow R$ defined by $f(x) = \sin x$, $\forall x \in R$ is one-one.

(b) The set of all odd integers forms a group with respect to addition.

(c) A pair on a Karnaugh map can eliminate one variable.

(d) Peterson graph is not Eulerian.

(e) The proposition $p \wedge p$ is equivalent to 1.

(iii) Fill in the blanks : **(5×1=5)**

(a) If $S = \{\phi, a\}$, then the set $S \cap P(S) =$ ________.

(b) The two types of quantifiers are ________.

(c) If for every element a in a group G, $a^2 = e$, then G is an ________ group.

(d) The sum of degrees of all vertices of a graph is equal to ________.

(e) A tree is a graph with no ________.

SECTION—B

2. Attempt any three parts of the following :— **(10×3=30)**

(a) If $f : A \rightarrow B$ and $g : B \rightarrow C$ be one-to-one onto function, then $g \circ f$ is also one-to-one onto and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(b) Show that the set of rational numbers Q forms a group under the binary operation $*$ defined by

$$a * b = a + b - ab, \ \forall \ a, b \in Q.$$

Is this group abelian ?

(c) Prove that the product of two lattices is a lattice.

(d) Construct the truth table for the following :

$$((P \to Q) \vee R) \vee (P \to Q \to R).$$

(e) Use generating function to solve the recurrence relation

$$a_{n+2} - 2a_{n+1} + a_n = 2^n$$

with conditions $a_0 = 2$, $a_1 = 1$.

SECTION—C

Note : Attempt all questions from this section, selecting any two parts from each question. **(5×2×5=50)**

3. (a) Prove that :

$$A - (B \cap C) = (A - B) \cup (A - C)$$

for all sets A, B and C.

(b) If I be the set of all integers and if the relation R be defined over the set I by xRy if $x - y$ is an even integer, where $x, y \in I$, show that R is an equivalence relation.

(c) Consider the functions $f, g : R \to R$, defined by

$$f(x) = 2x + 3 \text{ and } g(x) = x^2 + 1.$$

Find the composition function $(g \circ f)(x)$ and $(f \circ g)(x)$.

4. (a) How many generators are there of the Cyclic group of order 8 ?

(b) Define a commutative ring with unity.

(c) Show that if a, b are arbitrary elements of a group G, then $(ab)^2 = a^2 b^2$ if and only if G is abelian.

5. (a) Let X be the set of factors of 12 and let $\leq$ be the relation divider i.e. $x \leq y$ if and only if x/y. Draw the Hasse diagram of $(X, \leq)$

(b) Define a boolean function. For any x and y in a boolean algebra, prove that

$$(x + y)' = x' \cdot y'.$$

(c) Prove that the number of vertices having odd degree in a graph is always even.

6. (a) Prove that the following is a tautology :

$$A \vee (\overline{B \wedge C}) = (A \vee \overline{B}) \vee \overline{C}.$$

(b) Given the value of $p \to q$ is true. Determine the value of $\sim p \vee (p \leftrightarrow q)$.

(c) Negate the statement :

For all real x, if "$x > 3$", then "$x^2 > 9$".

7. (a) Find the generating function of the following numeric function :

$$a_n = \frac{1}{(n+1)!}, \ n \geq 0.$$

(b) State and prove Pigeonhole principle.

(c) Determine the numeric function for the corresponding generating function :

$$G(x) = \frac{10}{1-x} + \frac{12}{2-x}.$$